Noise Sharing and Mexican Hat Coupling in a Stochastic Neural Field

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A diffusion-type coupling operator biologically significant in neuroscience is a difference of Gaussian functions (Mexican Hat operator) used as a spatial-convolution kernel. We are interested in pattern formation by stochastic neural field equations, a class of space-time stochastic differential-integral equations using the Mexican Hat kernel. We explore, quantitatively, how the parameters that control the shape of the coupling kernel, coupling strength, and aspects of spatially-smoothed space-time noise, influence the pattern in the resulting evolving random field. We confirm that a spatial pattern that is damped in time in a deterministic system may be sustained and amplified by stochasticity. We find that spatially-smoothed noise alone causes pattern formation even without direct spatial coupling. Our analysis of the interaction between coupling and noise sharing allows us to determine parameter combinations that are optimal for the formation of spatial pattern.