On Kantorovitch type contrasts and surfaces for convex costs

Philippe Berthet*

* *University of Toulouse, France*

Abstract. We present results concerning the exact weak convergence of various random elements built from a collection of joint empirical quantiles, themselves generated simultaneously from one or more samples through a collection of mappings. In an univariate setting these random elements are Kantorovitch type contrasts on quantiles, for symmetric or asymmetric convex costs. They may also be viewed as weighted integrals of quantile transforms. In the multivariate setting they are closed random surfaces indexed by directions or transforms. The most classical special case is the Wasserstein distance between two empirical distribution functions from two dependent samples. Depending on the tails and cost, and under some conditions linking both aspects, different CLT's, weak convergence or exact escape rates are derived. The dependency typically affects the limiting variances, which are minimized under the copula of the optimal transportation, and sometimes even affects rates, so that the independence also plays a special role. The sharp study of these untrimmed integrals also provides a better understanding of the various bias phenomenons with respect to possibly desired levels of truncations. Technically we have to work on strong joint quantiles approximation probability spaces very closely to weak convergence requirements since some extreme or non extreme remainder terms do not even converge in probability. Applications of these Kantorovitch type contrasts and directional surfaces concern multiple tests, goodness of fit or comparison tests in multivariate or functional or process paths settings.

These results come from collaborations with J.C. Fort in a series of papers, the initial one being also with T. Klein (AIHP, 2019).