Complexity and dimension of finite-dimensional continuous maps

Boris Darkhovsky

Institute for Systems Analysis, FRC CSC RAS, Moscow, Russia

The concept of the ϵ-complexity of finite-dimensional continuous maps is proposed. This concept is in line with the Kolmogorov's idea on "complexity" of an object. Roughly speaking, the ϵ-complexity of an individual continuous map on a compact is the (logarithmic) number of the function values on a uniform grid that must be retained to reconstruct the map via a certain fixed family of approximation methods with a given relative error ϵ.

In other words, the ϵ-complexity can be called a *shortest description* (up to ϵ) of the map by its values on a unifrom grid with the help of given set of approximation methods.

It can be shown that the ϵ-complexity of "almost any" map satisfying Hölder condition is effectively characterized by couple of real numbers. We call these numbers the *ϵ-complexity coefficients*.

The main result of our ϵ-complexity theory is generalized to the case when a continuous map is given by its trace on some uniform grid.

Based on the concept of the ϵ-complexity, we propose a new variant of the dimension for the graph of a continuous map.

The ϵ-complexity is the foundation of *model-free* methodology of segmentation and classification for data of arbitrary (stochastic, deterministic or mixed) nature, in particular, for multidimensional time series.

Our model-free technology will be demonstrated on simulations and application to real data.