Rough calculus: pathwise calculus for functionals of irregular paths

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Abstract:

In his seminal paper *Calcul d'Ito sans probabilités* [7] Hans Föllmer showed that the Ito formula holds pathwise for functions of paths with finite quadratic variation along a sequence of partitions. Building on Föllmer's insight, we construct a pathwise calculus for smooth functionals of continuous paths with finite p-th variation along a sequence of time partitions, for arbitrary $p > 0$ [5]. We construct a pathwise integral, defined as a pointwise limit of compensated Riemann sums and show that the pathwise integral satisfies a change of variable formula and a pathwise isometry formula [1]. Results for functions are extended to path-dependent functionals using a concept of functional derivative introduced by Dupire [6]. As a result we obtain a unique signal plus noise decomposition for regular functionals of paths with strictly increasing p-th variation [1, 5]. Our results apply to sample paths of semi-martingales as well as fractional Brownian motion with arbitrary Hurst parameter $H > 0$ and other 'rough' processes. We show that the robustness of this construction with respect to the choice of the partition is linked to a roughness property of the underlying paths [3].

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