Change-point tests based on the Hill estimator for Long-Memory Stochastic Volatility Time Series

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Abstract. Let X_k, $k \in \mathbb{Z}$, be a stationary time series with tail index $\alpha > 0$, i.e. $\bar{F}(x) := P(X_1 > x) = x^{-\alpha}L(x)$, where L is a slowly varying function. Then, we have

$$\lim_{u\to\infty} \mathrm{E}\left(\log\left(\frac{X_1}{u}\right) \mid X_1 > u\right) = \frac{1}{\alpha} =: \gamma,$$

such that γ can be estimated by the Hill estimator

$$\hat{\gamma} := \frac{1}{\sum_{j=1}^{n} 1_{\{X_j > u_n\}}} \sum_{j=1}^{n} \log\left(\frac{X_j}{u_n}\right) 1_{\{X_j > u_n\}},$$

where u_n, $n \geq 1$, is a deterministic sequence with $u_n \to \infty$ and $n\bar{F}(u_n) \to \infty$. We consider a change-point test based on the Hill estimator to test for structural changes in the tail index of long-memory stochastic volatility (LMSV) time series. In order to determine the asymptotic distribution of the corresponding test statistic, we prove a uniform reduction principle for the tail empirical process in a two-parameter Skorohod space. It is shown that such a process displays a dichotomous behavior according to an interplay between the Hurst parameter, i.e. a parameter characterizing the dependence in the data, and the tail index. We will see that, nonetheless, long-memory does not have an influence on the asymptotic behavior of the test statistic.